12.802 Small-scale ocean dynamics

Problem set 1. Due date: March 4, 2019

1. **The full dispersion relationship of waves in a compressible-stratified fluid.**
   In class we derived the dispersion relationship for linear waves in a compressible, stratified fluid. The derivation assumed that the Coriolis frequency f was constant. Assume now that $f = f_0 + \beta y$ an approximation that retains the leading order effect of Earth's curvature. (Assume β is constant.) Derive the full dispersion relationship for this case- Show that the dispersion relationship has five non-trivial solutions. In particular derive an approximate expression for the smallest wave frequency in the limit $\beta \ll f\kappa_h$, where κ_h is the horizontal wavenumber of the waves. Do you recognize these waves. Why did we miss them in class by assuming $\beta = 0$?

2. **Newton's approach to waves.**
   Read Propositions XLIV-XLVI from Newton's Principia (pages 581 through 584 in Chandrasekhar's edition or the link below.) Recast Newton's argument in modern language by finding analytical expressions for the period of the pendulum and for the oscillations of the water in the canal. Discuss if the period of oscillations reflect deep or shallow gravity waves.

3. **Wave generation and propagation.**
   A storm out to sea generates short surface gravity waves which propagate away from the disturbance. Suppose you're sitting on an island watching the swell. At noon, the waves have a period of 10 seconds; at 7PM the period is 5 seconds. How far away was the storm?

   Hint: using the ideas from stationary phase, figure out how the wavenumber and frequency vary with time at a fixed observing point.

4. **Long surface waves.**
   Here is some data from the famous seismic sea wave of April 1, 1946. Suppose we regard the wave speed as fixed at some average depth $c = \sqrt{gH}$. Estimate the location of the earthquake, the time of occurrence, and the average depth H. The time of arrival is given in the table. Take advantage of the fact that the wave reached Crescent City and Honolulu almost simultaneously.

City	Lat	Lon	Arrival time (GMT)
Sitka	57° 3' 5.62"N	135° 20' 19.11"W	15:25
Crescent City	41° 45' 21"N	124° 12' 06"W	17:07
Honolulu	21° 18' N	157° 49' W	17:03
La Jolla	32° 50' 24"N	117° 16' 37"W	18:40
Valparaiso	33° 03' S	71° 37' W	30:36