12.802

Small Scale Ocean Dynamics

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Sound waves

The equations of motions for linear waves in a stratified fluid derived in the last class can be rearranged in the following way,

$$\nabla_h^2 \frac{\partial^2 \phi}{\partial t^2} + f^2 \nabla_h^2 \phi + \frac{1}{\rho_r} \nabla_h^2 \frac{\partial p}{\partial t} = 0, \tag{1}$$

$$\frac{\partial^2 w}{\partial t^2} + \frac{1}{\rho_r} \frac{\partial^2 p}{\partial z \partial t} + N^2 w = 0, \tag{2}$$

$$\frac{1}{c_s^2 \rho_r} \frac{\partial^2 p}{\partial t^2} + \frac{\partial}{\partial t} \nabla_h^2 \phi + \frac{\partial^2 w}{\partial z \partial t} = 0. \tag{3}$$

Sound waves have frequencies of order $\omega \gg N \geq f$ and thus the equations (1) through (3) are well approximated by,

$$\nabla_h^2 \frac{\partial^2 \phi}{\partial t^2} + \frac{1}{\rho_r} \nabla_h^2 \frac{\partial p}{\partial t} \simeq 0, \tag{4}$$

$$\frac{\partial^2 w}{\partial t^2} + \frac{1}{\rho_r} \frac{\partial^2 p}{\partial z \partial t} \simeq 0, \tag{5}$$

$$\frac{1}{c_s^2 \rho_r} \frac{\partial^2 p}{\partial t^2} + \frac{\partial}{\partial t} \nabla_h^2 \phi + \frac{\partial^2 w}{\partial z \partial t} = 0. \tag{6}$$

Using (4) and (5) to reduce (6) to an equation for pressure we have,

$$\frac{1}{c_s^2} \frac{\partial^2 p}{\partial t^2} + \nabla_h^2 p + \frac{\partial^2 p}{\partial z^2} = 0, \tag{7}$$

where we ignored the compressive term in the vertical momentum equation and vertical variations in both ρ_r and $c_s = c_s(\theta_r, S_r, p_r)$. These terms become important only on vertical scales of order c_s^2/g, $\rho_r/\partial\rho_r/\partial z$, $c_s/\partial c_s/\partial z$ all of which are deeper than the full ocean depth.

Substituting wave solutions in (7), we recover the dispersion relationship for sound waves derived in the last lecture, if c_s is constant,

$$\omega^2 = c_s^2 \kappa^2.$$

Sound waves are non dispersive. The group velocity is in the direction of the wavenumber $\boldsymbol{k}$, $\boldsymbol{c}_g = c_s \boldsymbol{k}/k$.

The wave guide

Sound travels about 1500 meters per second in seawater, much faster than in air where the sound speed is about 340 meters per second. Notice however that the speed of sound in seawater is not a constant value. It varies by a small amount (a few percent) from place to place, season to season, morning to evening, and with water depth. Although the variations in the speed of sound are not large, they have important effects on how sound travels in the ocean.

Sound speed is affected by temperature, salinity, and pressure. Here we are referring to the ocean pressure due to the weight of the overlying water (equilibrium pressure), not to the pressure fluctuations associated with a sound wave, which are much, much smaller. The speed of sound in water increases with increasing water temperature, increasing salinity and increasing pressure (depth). The approximate change in the speed of sound with a change in each property is 4 m/s for a change of 1 celsius degree, 1.4 m/s for a salinity change of 1 psu, and 17 m/s for a depth (pressure) change of 1000 m.

In mid-latitudes temperature decreases rapidly with depth in the upper ocean and as a result the sound speed decreases from the surface. Below the mid-latitude ocean thermoclines temperature changes are small (the temperature decrease from its surface value to about 2 degrees in the upper 1000 m) and the sound speed increases with depth due to increase in pressure. Because of the competing effects of temperature and pressure, the sound speed reaches a minimum at roughly 1000 meter depth in mid-latitudes. This minimum creates a sound channel that lets sound travel long distances in the ocean.

We can study the impact of the minimum in sound speed on sound wave propagation in the ocean by studying the ray equation for sound waves following the WKB approach outlined by Glenn in the study of surface gravity waves. An approximate

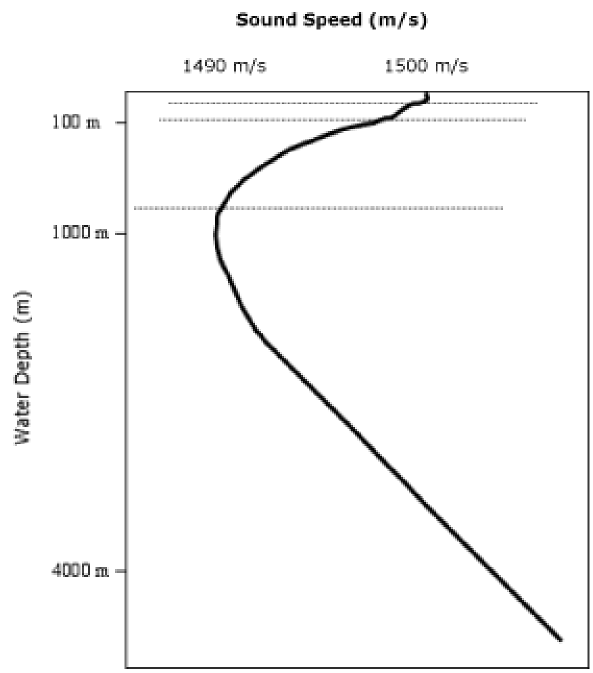

Figure 1: Typical sound speed profile as a function of depth in the midlatitude ocean.

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solution for the equation of sound rays is obtained for small vertical excursions around the sound minimum velocity (i.e. $m \ll \kappa_h$), whereas the 3D wavenumber varies only slightly. With the parabolic sound velocity profile,

$$c_s(z) = c_{min} + \frac{1}{2}\gamma(z - z_{min})^2,$$

we can solve the evolution of the wavenumbers along the ray paths,

$$\begin{aligned} \left(\frac{\partial}{\partial t} + \boldsymbol{c}_g \cdot \nabla\right) k_h &= -\frac{\partial c_s(z)\kappa_h(z)}{\partial x} = 0, \\ \left(\frac{\partial}{\partial t} + \boldsymbol{c}_g \cdot \nabla\right) m &= -\frac{\partial c_s(z)\kappa_h(z)}{\partial z} \simeq \kappa_h(z_{min})\frac{\partial c_s(z)}{\partial z} = -\gamma\kappa(z_{min})(z - z_{min}). \end{aligned}$$

The ray equations are $\dot{\boldsymbol{x}} = \boldsymbol{c}_g$. The horizontal ray path is therefore,

$$\dot{x}_h = c_s(z)\frac{k_h}{\kappa(z)} \approx c_s(z_{min})\frac{k_h}{\kappa(z_{min})} \approx c_{min},$$

remembering that k_h is constant. For the vertical component of the wave path,

$$\dot{z} = c_s(z)\frac{m}{\kappa(z)} \simeq c_{min}\frac{m}{\kappa(z_{min})}.$$

Combining the equation for the vertical ray path with the equation for the vertical wavenumber , we can derive the equation for the evolution of the sound ray about the sound minimum velocity. Introducing $\zeta = z - z_{min}$,

$$\ddot{\zeta} = c_{min} \frac{\dot{m}}{\kappa(z_{min})} \quad \Longrightarrow \quad \ddot{\zeta} + \gamma c_{min} \zeta = 0. \tag{8}$$

This equation has oscillatory solutions. The rays undergo an oscillatory path with an oscillation period $T = 2\pi/\sqrt{\gamma c_{min}}$. As the horizontal propagation velocity is approximately given by c_{min}, this corresponds to a horizontal length scale $L \approx c_{min} T = 2\pi\sqrt{c_{min}/\gamma}$. With characteristic values of $\gamma \approx 2 \times 30$ m/s $/(1,000\text{ m})^2 \approx 0.6 \times 10^{-4}$ m^{-1}s^{-1} and $c_{min} \approx 1550$ m s^{-1}, one obtains $L \approx 30$ km for the distance between two maxima (or minima) of the ray curves and periods of 25 s, consistent with observations.

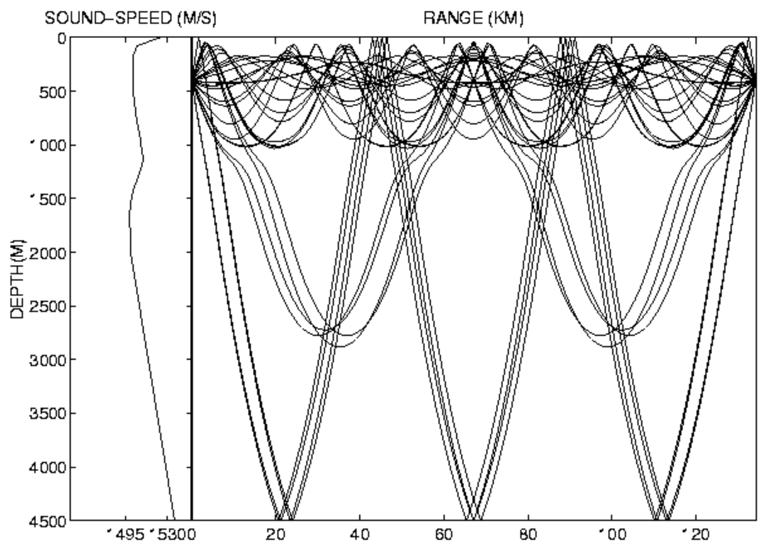

Figure 2: Typical sound speed profile and ray tracing from the Bay of Biscay.

Seismic oceanography

Seismic oceanography (SO) is a new technique that exploits the reflection of sound waves by changes in sound speed profiles due to changes in temperature and salinity. The idea is to sample the wave reflections to reconstruct the temperature and salinity fluctuations. SO was born out of the accidental discovery that seismic systems designed to image sub-seafloor geologic structures, such as hydrocarbon traps, were also providing images of water layers in the ocean. Seismic reflections from water layers of different densities (due to temperature and salinity variations) are about 1000 times weaker than reflections from sediment interfaces, and for decades geophysicists had been looking right through them without ever noticing them. The reflections require careful processing of high-quality seismic data to see and interpret, but can show subtle details in the water mass boundaries that occur on lateral scales of only a few meters. Standard wire-line oceanography measurements, such as expendable bathythermograph (XBT) or conductivity-temperature-depth (CTD) casts, typically have measurement spacing of 1 to 10 kilometers; so the seismic oceanographic records with measurements every 5 to 10 meters, both horizontally and vertically, allow measurement of the ocean water masses at horizontal resolutions rarely seen. Because of the aspect ratio of the ocean, obtaining oceanographic measurements at similar resolutions both vertically and horizontally is very unusual, and so represents a new way of observing ocean phenomena.

SO can be thought of as a type of acoustic oceanography (AO) but it differs in a very important way. The lower frequency sound waves used in SO, about 10 to 200 Hz, are coherently reflected (not refracted or incoherently scattered) directly by the thermohaline boundaries between water masses. These frequencies are affected by vertical changes on the scale of meters to tens of meters. Conversely, the sound waves commonly used in AO are at frequencies about 1000 times higher, 10 to 200 kHz, and are affected by features on the scale of microns to millimeters, such as suspended impurities or biota in the water column. Although the scatterers seen in AO records (e.g., from acoustic Doppler current profilers, ADCPs) may be associated with physical boundaries, the AO records do not contain the information to quantify the magnitude of physical property contrasts between water mass boundaries as do SO records.

The acquisition geometry for SO requires a ship, a sound source and a array of hydrophones. A ship moving at 4 to 6 knots tows a controlled sound source and a linear array of tens to thousands of hydrophones, both at a depth of a few meters. The sound source is typically an air gun, a device that explosively releases compressed air, creating a sound pulse in the ocean with a duration of 10 to 20 milliseconds. The sound pulse reflects off of near-horizontal interfaces such as sediment layers in the earth or vertical sound speed changes in the ocean, and is then recorded at the hydrophone array. Typically, the source is fired every 25 meters (about every 12 seconds). The hydrophone array contains 10 to 20 individual hydrophones that are summed to generate a single channel record, and may contain dozens to hundreds of channels, with each one providing an independent measurement of ocean reflectivity.

Read "Thermohaline Fine Structure in an Oceanographic Front from Seismic Reflection Profiling" by Holbrook, Páramo, Pearse, and Schmitt published in Science in 2003.

Filtering out sound waves

In the previous lecture we shoed that linear waves in a compressible ocean satisfy the set of equations,

$$\nabla_h^2 \frac{\partial \psi}{\partial t} + f \nabla_h^2 \phi = 0, \tag{9}$$

$$\nabla_h^2 \frac{\partial \phi}{\partial t} - f \nabla_h^2 \psi + \frac{1}{\rho_r} \nabla_h^2 p = 0, \tag{10}$$

$$\frac{\partial w}{\partial t} + \frac{1}{\rho_r} \frac{\partial p}{\partial z} - b = 0, \tag{11}$$

$$\frac{1}{c_s^2 \rho_r} \frac{\partial p}{\partial t} + \nabla_h^2 \phi + \frac{\partial w}{\partial z} = 0, \tag{12}$$

$$\frac{\partial b}{\partial t} + N^2 w = 0, \tag{13}$$

Neglecting the time dependence in equation (12) is the most common approximation in oceanic studies,

$$\nabla_h^2 \frac{\partial \psi}{\partial t} + f \nabla_h^2 \phi = 0, \tag{14}$$

$$\nabla_h^2 \frac{\partial \phi}{\partial t} - f \nabla_h^2 \psi + \frac{1}{\rho_r} \nabla_h^2 p = 0, \tag{15}$$

$$\frac{\partial w}{\partial t} + \frac{1}{\rho_r} \frac{\partial p}{\partial z} - b = 0, \tag{16}$$

$$\cancel{\frac{1}{c_s^2 \rho_r} \frac{\partial p}{\partial t}} + \nabla_h^2 \phi + \frac{\partial w}{\partial z} = 0, \tag{17}$$

$$\frac{\partial b}{\partial t} + N^2 w = 0, \tag{18}$$

The dispersion relations reduces to,

$$\omega^3 \kappa^2 - \omega(\kappa_h^2 N^2 + m^2 f^2) = 0, \tag{19}$$

which includes only gravity and "Rossby" waves. Hence the neglect of the time dependence of pressure on density, which gives the time derivative term in equation (12), is sufficient to remove the sound waves completely from the system, without significant distortion of either the gravity or Rossby wave modes.